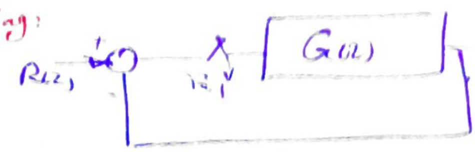
Sheet 4.

2 For the system shown in figure below with discrete transfer function

$$G(z) = \frac{0.1(z+0.9)}{(z-1)(z-0.7)}$$

Determine whether the system is stable or not using:

1. Bilinear Transformation
2. Jury Test



Sol^n

$$C.L.T.F. = \frac{C(z)}{R(z)} = \frac{G(z)}{1+G(z)} \Longrightarrow \text{c/c eqn} \quad 1+G(z)=0$$

$$\text{c/c eqn} = 1 + \frac{0.1(z+0.9)}{(z-1)(z-0.7)} = 0$$

$$(z-1)(z-0.7) + 0.1(z+0.9) = 0$$

$$z^2 - 1.6z + 0.79 = 0$$

① Bilinear Transformation,

$$z = \frac{1+r}{1-r}$$

$$\because P(z) = z^2 - 1.6z + 0.79 = 0 \qquad \text{c/c eqn}$$

$$\left(\frac{1+r}{1-r}\right)^2 - 1.6\left(\frac{1+r}{1-r}\right) + 0.79 = 0$$

$$(1+r)^2 - 1.6(1+r)(1-r) + 0.79(1-r)^2 = 0$$

$$1 + 2r + r^2 - 1.6(1-r^2) + 0.79(1-2r+r^2) = 0$$

$$3.39r^2 + 0.42r + 0.19 = 0$$

Construct Routh Array

$$\begin{array}{c|cc} r^2 & 3.39 & 0.19 \\ r & 0.42 & \\ r^0 & 0.19 & \end{array}$$

There is no sign change in first column of the array

$\Downarrow$

Stable System

2 Jury test

$$P(z) = Z^2 - 1.6Z + 0.79$$

1. $P(1) = (1)^2 - 1.6(1) + 0.79 > 0$ ✓
2. $(-1)^2 P(-1) = (-1)^2 - 1.6(-1) + 0.79 > 0$ ✓
3. $|a_0| = 0.79 \leqslant |a_2| = 1$ ✓

Stable System

So For 2nd order System there is no need to find b_0, b_n and other elements;

3 Another Way (Poles Locations)

we can determine stability directly from Poles locations

1. stable system if all Poles inside the unit Circle
2. Critical Stable System if some poles on the Circumference of unit Circle
3. Unstable System if one Pole at least outside the unit Circle

c/c eqn $\quad Z^2 - 1.6Z + 0.79 = 0$

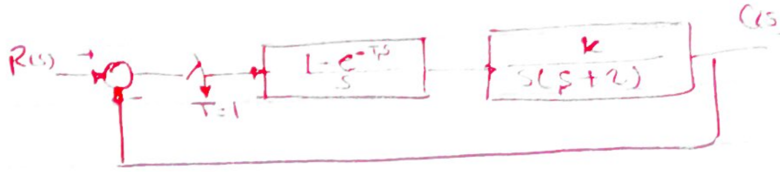
$$Z_{1,2} = \frac{1.6 \pm \sqrt{(-1.6)^2 - 4 * 0.79}}{2} = 0.8 \pm j0.387$$

$$|Z_{1,2}| = \sqrt{0.8^2 + 0.387^2} = 0.889 < 1$$

Stable System

4 Find Range of k for stability

a) Solution



$$G(z) = Z\left[\frac{1-e^{-Ts}}{s} \cdot \frac{k}{s(s+2)}\right]$$

$$= k(1-z^{-1})\, Z\left[\frac{1}{s^2(s+2)}\right] = k(1-z^{-1})\, Z\left[\frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+2}\right]$$

$A_1 = 0.5$, $A_3 = 0.25$ (By inspection)

To find A_2

let $s=1 \longrightarrow \frac{1}{3} = 0.5 + \frac{0.25}{3} + A_2 \longrightarrow A_2 = -0.25$

$$G(z) = k\left(\frac{z-1}{z}\right) Z\left[\frac{0.5}{s^2} - \frac{0.25}{s} + \frac{0.25}{s+2}\right] = k\frac{(z-1)}{z}\left[\frac{0.5z}{(z-1)^2} - \frac{0.25z}{z-1} + \frac{0.25z}{z-e}\right.$$

$$= k\left[\frac{0.5}{(z-1)} - 0.25 + \frac{0.25(z-1)}{z-0.135}\right]$$

$$= k\left[\frac{0.5(z-0.135) - 0.25(z-1)(z-0.135) + 0.25(z-1)^2}{(z-1)(z-0.135)}\right]$$

$$= k\left[\frac{0.5z - 0.0677 - 0.25(z^2 - 1.135z + 0.135) + 0.25(z^2 - 2z + 1)}{(z-1)(z-0.135)}\right]$$

$$= k\left[\frac{0.284z + 0.149}{(z-1)(z-0.135)}\right]$$

c/c eqn $\longrightarrow 1 + G(z) = 0$

$$1 + \frac{k(0.284z + 0.149)}{(z-1)(z-0.135)} = 0$$

$$(z-1)(z-0.135) + k(0.284z + 0.149) = 0$$

$$z^2 + (0.284k - 1.135)z + (0.149k + 0.135) = 0$$

Using Jury test :

For stable system

① $P(1) > 1$

$\therefore \cancel{1} + 0.284K \cancel{-1.135} + \cancel{0.135} + 0.149k > 0 \longrightarrow \boxed{k > 0}$

② $(-1)^2 P(-1) > 1$

$\therefore 1 - 0.284K + 1.135 + 0.135 + 0.149k > 0$

$2.27 - 0.135k > 0$

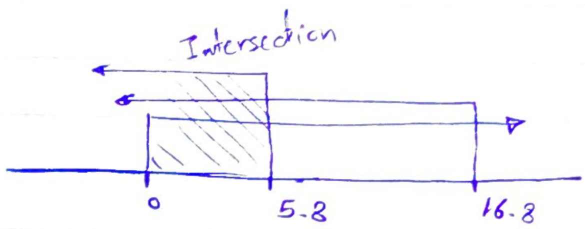
$\boxed{K < 16.8}$

③ $|a_0| < |a_2|$

$0.135 + 0.149k < 1$

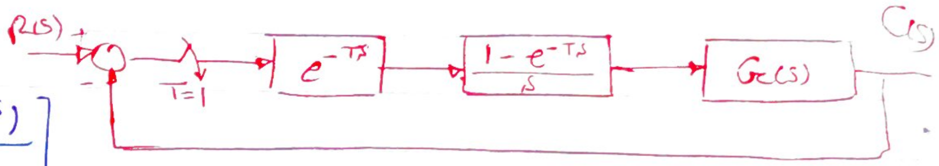
$0.149K < 0.865$

$\boxed{K < 5.8}$



$\therefore$ The Range of K for Stability $0 < K < 5.8$

b) solution



$G(S) = \frac{K}{S(S+1)}$

$$G(z) = Z\left[\frac{Ke^{-TS}(1-e^{-TS})}{S^2(S+1)}\right]_{T=1}$$

$$= K(1-Z^{-1})Z^{-1}\, Z\left[\frac{1}{S^2(S+1)}\right]$$

$$= \frac{K(Z-1)}{Z^2}\, Z\left[\frac{1}{S^2} - \frac{1}{S} + \frac{1}{S+1}\right] = \frac{K(Z-1)}{Z^2}\left[\frac{Z}{(Z-1)^2} - \frac{Z}{Z-1} + \frac{Z}{Z-e^{-1}}\right]$$

$$= \frac{K}{Z}\left[\frac{1}{Z-1} - 1 + \frac{(Z-1)}{Z-0.368}\right] = \frac{K\{Z-0.368-(Z-1)(Z-0.368)+(Z-1)^2\}}{Z(Z-1)(Z-0.368)}$$

$$= \frac{K(Z-0.368-\cancel{Z^2}+1.368Z-0.368+\cancel{Z^2}-2Z+1)}{Z(Z-1)(Z-0.368)} = \frac{K(0.368Z+0.264)}{Z(Z-1)(Z-0.368)}$$

C/C eqn $1 + G(z) = 0$

$$1 + \frac{K(0.368z + 0.264)}{z(z-1)(z-0.368)} = 0$$

$$z(z-1)(z-0.368) + K(0.368z + 0.264) = 0$$

$$z^3 - 1.368z^2 + (1+K)(0.368z) + 0.264K = 0 \longrightarrow \text{C/C eqn}$$

To check Stability for higher order Systems use Bilinear Transformation $z = \frac{1+r}{1-r}$

$$\therefore \left(\frac{1+r}{1-r}\right)^3 - 1.368\left(\frac{1+r}{1-r}\right)^2 + 0.368(1+K)\left(\frac{1+r}{1-r}\right) + 0.264K = 0$$

$$(1+r)^3 - 1.368(1+r)^2(1-r) + 0.368(1+K)(1+r)(1-r)^2 + 0.264K(1-r)^3 = 0$$

$$1 + 3r + 3r^2 + r^3 - 1.368(1 + r - r^2 - r^3) + 0.368(K+1)(1 - r - r^2 + r^3) + 0.264K(1 - 3r + 3r^2 - r^3) = 0$$

$$(1 + 1.368 + 0.368 + 0.368K - 0.264K)r^3 + (3 + 1.368 - 0.368 - 0.368K + 0.792K)r^2 + (3 - 1.368 - 0.368K - 0.368 - 0.792K)r + (1 - 1.368 + 0.368K + 0.368 + 0.264K) = 0$$

$$(2.736 + 0.104K)r^3 + (4 + 0.424K)r^2 + (1.264 - 1.16K)r + 0.632K = 0$$

Routh Array

r^3	$2.736 + 0.104K$	$1.264 - 1.16K$
r^2	$4 + 0.424K$	$0.632K$
r^1	α	
r^0	$0.632K$	

$$\alpha = \frac{(4+0.424K)(1.264-1.16K) - 0.632K(2.736+0.104K)}{4+0.424K}$$

$$= \frac{-0.558K^2 - 5.833K + 5.056}{4+0.424K}$$

For the System to be Stable all elements of the 1st column in Routh array must be Larger than Zero (No Change in Sign)

① $2.736 + 0.104K > 0 \longrightarrow \boxed{K > -26.31}$

② $4 + 0.424K > 0 \longrightarrow \boxed{K > -9.434}$

③ $\alpha > 0$

$$-0.558K^2 - 5.833K + 5.056 > 0$$

$$0.558K^2 + 5.833K - 5.056 < 0$$

$$K^2 + 10.45K - 9.061 < 0$$

↳ the roots of eqn $K_{1,2} = \frac{-10.45 \pm \sqrt{(10.45)^2 + 4(9.061)}}{2} = 0.805 \;\&\; -11.26$

$$(K - 0.805)(K + 11.26) < 0$$

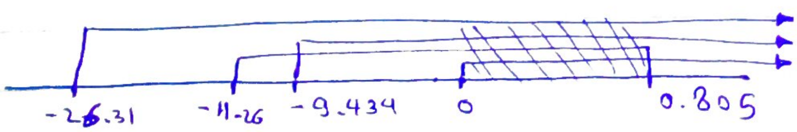
$K < 0.805$ and $K > -11.26$

$K > 0.805$ and $K < -11.26$ illogical → refused

$\therefore \boxed{-11.26 < K < 0.805}$

④ $\boxed{K > 0}$

The intersection Between all Conditions



the Range of K for stability

$$\boxed{0 < K < 0.805}$$